

PHYSICS OF INDUCTION COOKING

Solution of the Experimental Problem

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1. EXP#1: CHARACTERIZATION OF THE INDUCTION COIL

1.1 Series RLC circuit

To determine L using resonance experiment we setup a series R - L - C circuit as shown below. We measure the source voltage from the Function Generator (FG): V_S and measure the current I by measuring the voltage across “shunt resistance” $R_1=1\ \Omega$: $I = V_{R1} / R_1$.

We use the digital oscilloscope to measure the voltage. For convenient and quick measurement, we can fix one probe terminal (e.g.) on node #1 and repetitively measure voltage on node#2 and node#3 to measure source voltage V_S and V_{R1} repetitively.

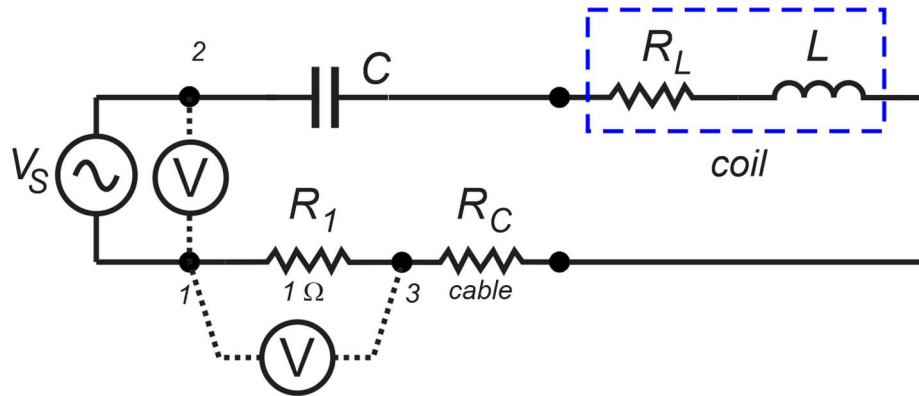


Figure 1. The series RLC circuit

The cables involved in the circuit are two pieces of banana-to-pins cable (item#6). Using ohmmeter we obtain: $R_C = 0.09\ \Omega$.

1.2 Resonance series RLC circuit

Then we determine the resonant condition where the impedance $Z = V_S / I$ reaches minimum or conductance $G = I / V_S$ reaches maximum. A smart student should quickly scan the frequency first to quickly find the resonant condition and suitable frequency range before taking data, e.g. fix V_S and scan I as a function of frequency. We note that V_S could vary due changing load impedance. The data is shown below:

SOLUTION

	C(Y)	f(X)	VS(Y)	VR1(Y)	G(Y)
Name					
Units	μF	kHz	V	V	S
Comments			(Vmax)	(Vmax)	I/VS
1	0.47	16.3	11.987	0.746	0.06223
2		21.6	9.133	0.967	0.10588
3		23.9	4.376	0.616	0.14077
4		28.4	1.465	0.373	0.25461
5		31.6	0.799	0.327	0.40926
6		32.5	0.742	0.32	0.43127
7		33	2.359	1.187	0.50318
8		33.8	2.283	1.157	0.50679
9		35.3	0.685	0.335	0.48905
10		35.6	2.436	1.149	0.47167
11		36.7	1.351	0.544	0.40266
12		37.5	0.894	0.32	0.35794
13		38.8	1.047	0.316	0.30181
14		40.6	1.332	0.335	0.2515
15		43.2	1.922	0.38	0.19771
16		48	3.235	0.464	0.14343
17		54.1	7.915	0.822	0.10385
18		64.8	4.719	0.373	0.07904

	C(Y)	f(X)	VS(Y)	VR1(Y)	G(Y)
Name					
Units	μF	Hz	V	V	S
Comments			(Vmax)	(Vmax)	I/VS
1	2200	20	4.034	0.936	0.23203
2		31	5.099	1.674	0.3283
3		53	5.023	2.207	0.43938
4		79	4.833	2.474	0.5119
5		109	4.795	2.55	0.5318
6		228	4.643	2.835	0.6106
7		282	4.643	2.892	0.62287
8		305	4.643	2.852	0.61426
9		328	4.643	2.854	0.61469
10		369	4.643	2.854	0.61469
11		463	4.567	2.854	0.62492
12		570	4.567	2.816	0.6166
13		710	4.49	2.816	0.62717
14		877	4.414	2.74	0.62075
15		1070	4.338	2.664	0.61411
16		1390	4.186	2.55	0.60917
17		1730	4.034	2.397	0.5942
18		2170	3.844	2.207	0.57414
19		3030	3.539	1.903	0.53772
20		4360	3.254	1.522	0.46773

Table 1. RLC resonance data for $C=470 \text{ nF}$ and $C=2200 \mu\text{F}$

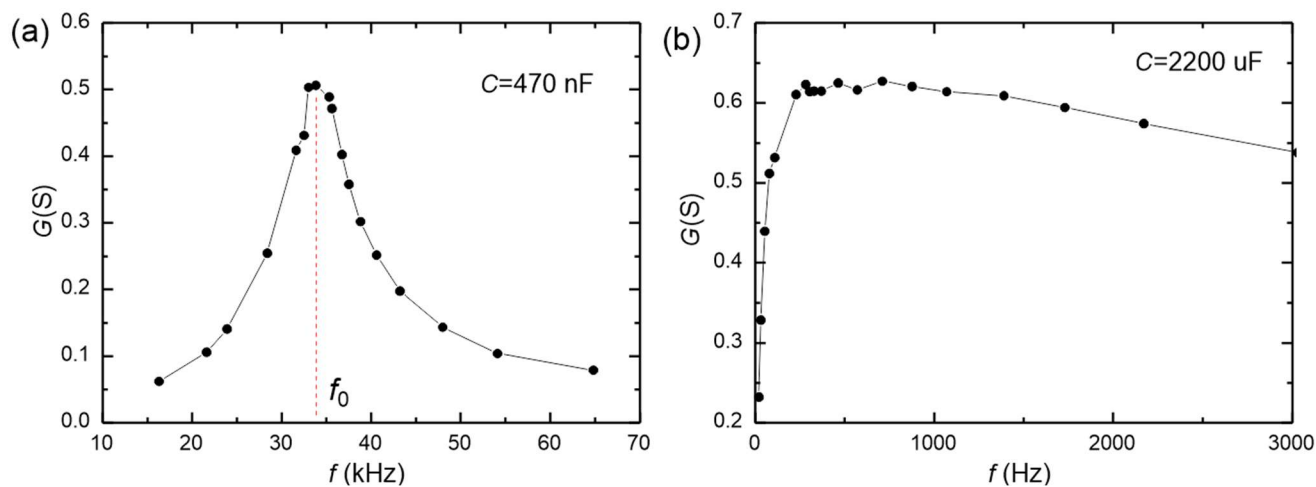


Figure 2. Resonance conductance plot of the RLC circuit with: (a) $C = 470 \text{ nF}$, (b) $C = 2200 \mu\text{F}$.

	C(X)	f0(Y)	L(Y)
Units	μF	Hz	μH
Comments		resonance	
1	0.47	33800	47.17
2	2200	400	71.96

Table 2. Results of L determination from RLC resonance

SOLUTION

The results are shown in Table 2. The resonance frequency is given as: $\omega_0 = 1/\sqrt{LC}$, thus $L = 1/\omega_0^2 C$. We note that the resonance data for $C = 470$ nF is nice and sharp and yields correct value of $L = 47.2$ μ H, while the data for $C = 2200$ μ F shows broad and poor resonance thus yield inaccurate value of $L = 72$ μ H. This happens because for a series RLC circuit the quality (Q) factor is given as: $Q = \sqrt{L/C}/R$, therefore smaller capacitance yields a higher quality factor or sharper resonance curve.

1.3 Alternative model to extract L and R_L

We can formulate the impedance as:

$$Z = R_T + j(\omega L - 1/\omega C), \quad (1)$$

$$Z^2 = R_T^2 + (\omega L - 1/\omega C)^2 = R_T^2 + \omega^2 L^2 - 2L/C + 1/\omega^2 C^2, \quad (2)$$

$$Z^2 - 1/\omega^2 C^2 = (R_T^2 - 2L/C) + \omega^2 L^2 \quad (3)$$

where the total resistance is: $R_T = R_1 + R_C + R_L$, with R_L is the coil internal resistance. We can linearize the last equation as: $y = a + bx$, where: $y = Z^2 - 1/\omega^2 C^2$, $x = \omega^2$, $b = L^2$ and $a = R_T^2 - 2L/C$. Note that since the inductor impedance dominate at high frequency we can also ignore the capacitance term: $1/\omega^2 C^2$ in the analysis.

We can solve for L and R_L as:

$$L = \sqrt{b} \quad (4)$$

$$R_L = \sqrt{a + 2L/C} - R_1 - R_C \quad (5)$$

1.4 RLC experiment to extract L and R_L with $C=470$ uF and 1000 uF

We perform RLC experiments for $C = 470$ μ F and 1000 μ F:

	C(Y)	f(X1)	VS(Y1)	VR1(Y1)	w2(X2)	y(Y2)
ame						
Units	uF	Hz	V	V	(rad/s)^2	Ohm
ents			(Vmax)	(Vmax)	w^2	Z-1/(w^2C^2)
1	470	481	7.516	3.844	9.134E+06	3.3274
2		635	7.326	3.996	1.592E+07	3.07673
3		810	7.23	4.034	2.590E+07	3.03744
4		946	7.04	4.034	3.533E+07	2.91747
5		1090	6.945	3.958	4.690E+07	2.98237
6		1280	6.65	3.844	6.468E+07	2.9228
7		1500	6.469	3.729	8.883E+07	2.9585
8		2140	5.994	3.311	1.808E+08	3.25225
9		2550	5.708	3.082	2.567E+08	3.41243
10		3220	5.233	2.664	4.093E+08	3.84757
11		3810	4.947	2.474	5.731E+08	3.99048

	C(Y)	f(X1)	VS(Y1)	VR1(Y1)	w2(X2)	y(Y2)
ame						
Units	uF	Hz	V	V	(rad/s)^2	Ohm
ents			(Vmax)	(Vmax)	w^2	Z-1/(w^2C^2)
1	1000	362	7.154	4.172	5.173E+06	2.74712
2		463	7.116	4.11	8.463E+06	2.87954
3		538	7.04	4.11	1.143E+07	2.8465
4		649	6.964	4.11	1.663E+07	2.81087
5		761	6.866	4.072	2.286E+07	2.79936
6		839	6.812	4.034	2.779E+07	2.81554
7		959	6.736	3.958	3.631E+07	2.86882
8		1270	6.507	3.805	6.367E+07	2.9088
9		1860	6.013	3.387	1.366E+08	3.14443
10		2550	5.518	2.968	2.567E+08	3.4526
11		3760	4.871	2.359	5.581E+08	4.26185
12		4690	4.567	2.055	8.684E+08	4.93784

Table 3. RLC measurements for $C = 470$ μ F and 1000 μ F

SOLUTION

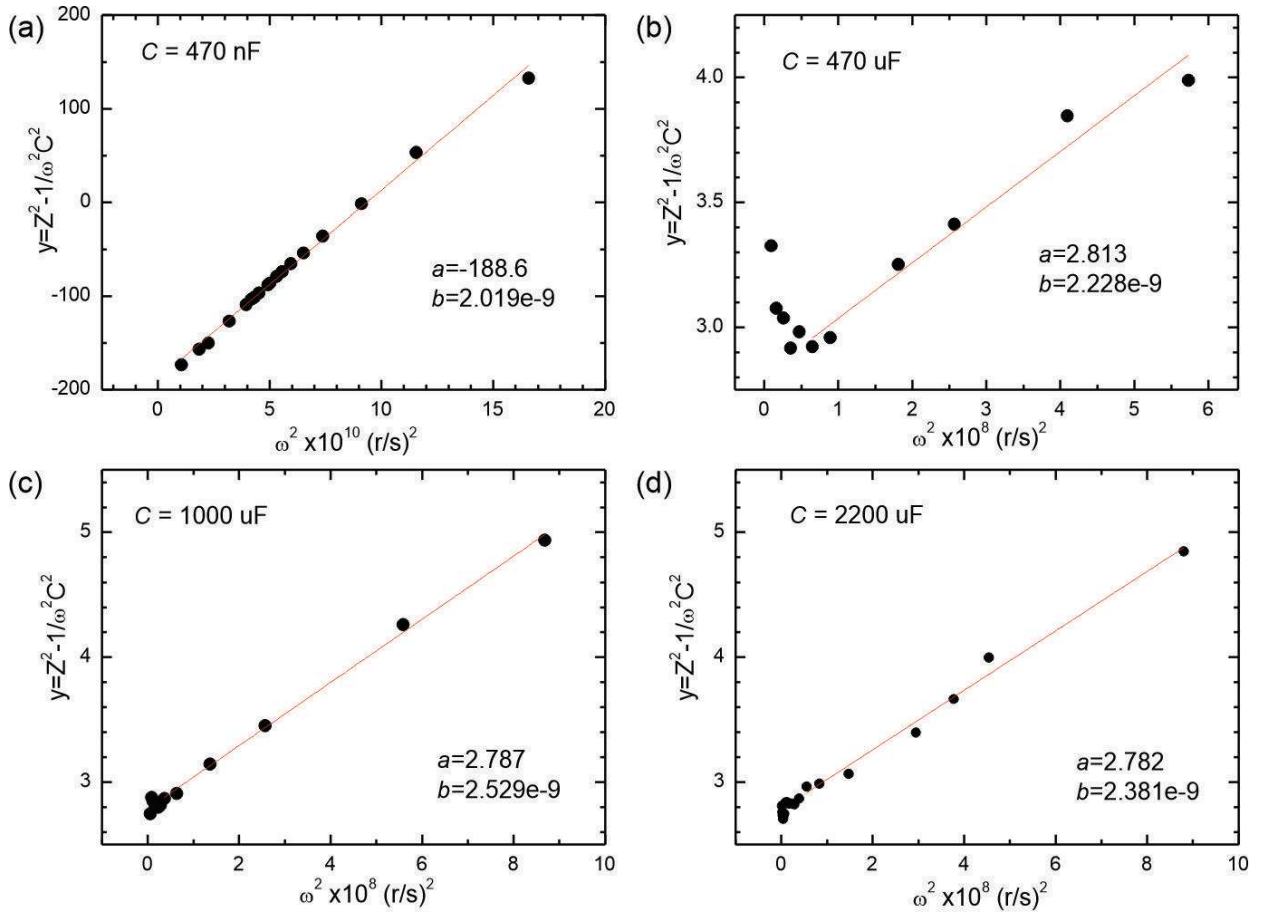


Figure 3. Extraction of R_L and L with $C = 470 \text{ nF}$, 470 uF , 1000 uF and 2200 uF .

Here are the results:

	C(X)	a(Y)	b(Y)	L(Y)	RL(Y)
Units	uF	Ohm^2	H^2	uH	Ohm
Results					
1	0.47	-188.6	2.019E-9	44.93	0.524
2	470	2.813	2.228E-9	47.20	0.646
3	1000	2.787	2.529E-9	50.29	0.609
4	2200	2.503	2.737E-9	52.32	0.507

Table 4. Results of L and R_L determination

We have average: $L = 48.7 \text{ uH}$ and average coil resistance: $R_L = 0.57 \text{ } \Omega$. This is consistent with the original specification of the coil (Würth Elektronik 760308101303): $L = 47 \text{ uH}$ and $R_L = 0.46 \text{ } \Omega$.

Therefore this second technique is more accurate in determining L even in the case where the resonance is poor. We obtain R_L as a “bonus” from the analysis as it comes from the linear fit “intercept” however they are less accurate.

SOLUTION

Measurement of R_L directly with the multimeter for verification is also acceptable, we obtain:

$$R_L = (0.47 \pm 0.03) \Omega .$$

2. EXP#2: MUTUAL INDUCTION AND SKIN DEPTH

2.A Mutual Induction

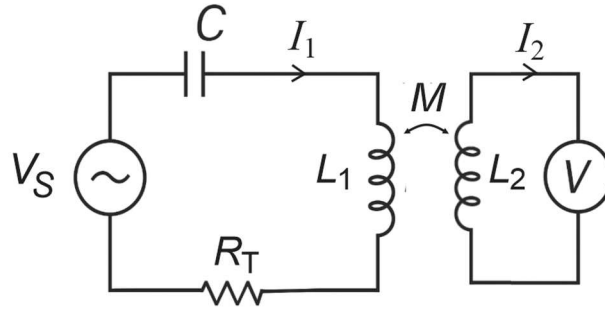


Figure 4. Mutual inductance setup

2.2 Mutual inductance experiment

Here we perform the measurement twice where the primary is coil#1 and secondary is coil#2 and then we swap them.

$$V_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} = -M \frac{di_1}{dt} \quad (6)$$

$$Z' = \frac{V_2}{I_1} = -j\omega M \quad (7)$$

The self inductance contribution is negligible since the second coil is connected to voltmeter and $i_2 \sim 0$. Then we can tabulate the “impedance” $Z = V_2 / I_1$ as a function of frequency and extract M .

	f(X)	I1(Y)	V2(Y)	Z(Y)		f(X)	I2(Y)	V1(Y)	Z(Y)
Units	Hz	A	V	Ohm	Units	Hz	A	V	Ohm
ments					ments				
1	1020	3.539	0.096	0.02713	1	1010	3.52	0.136	0.03864
2	1990	3.082	0.184	0.0597	2	2000	3.006	0.2	0.06653
3	3000	2.588	0.241	0.09312	3	3000	2.512	0.27	0.10748
4	4040	2.131	0.278	0.13046	4	4000	2.112	0.299	0.14157
5	5010	1.827	0.295	0.16147	5	5000	1.808	0.316	0.17478
6	6000	1.617	0.312	0.19295	6	6060	1.56	0.337	0.21603
7	7010	1.427	0.32	0.22425	7	7020	1.355	0.339	0.25018
8	8060	1.256	0.342	0.27229	8	8060	1.21	0.358	0.29587
9	9140	1.104	0.35	0.31703	9	9010	1.104	0.369	0.33424
10	10100	1.012	0.358	0.35375	10	10200	0.997	0.373	0.37412

Table 5. Mutual inductance determination

SOLUTION

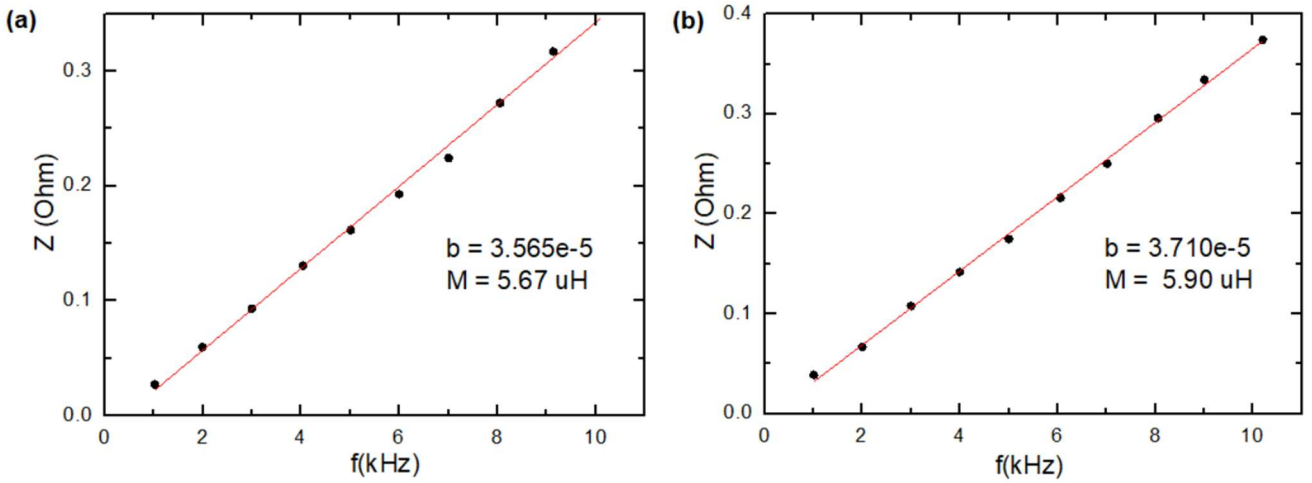


Figure 5. Mutual inductance determination results: (a) Primary=Coil#1, Secondary=Coil#2 (b) Primary=Coil#2, Secondary=Coil#1.

2.3 Mutual inductance results

We fit the data to linear equation: $y = a + bx$. Here we obtain the mutual inductance: $M = b / 2\pi$, and we obtain M reasonably close results between the two measurements: $M_{1-2} = 5.67 \mu\text{H}$ and $M_{2-1} = 5.90 \mu\text{H}$ with average: $M = 5.79 \mu\text{H}$.

2.B Skin depth experiment

2.4 Skin depth equations model and experiments

Equation model to determine n :

We can perform two-step linear regressions to extract n and σ as follows. For each metal we perform measurement of the voltage in secondary coil (coil#1) which is proportional to B after it passes through the metals:

$$V_2 \sim B(z) = B_0 \exp(-z / \delta) = B_0 \exp(-N t_0 / \delta(f)) \quad (8)$$

where t_0 is the metal thickness and N is the number of metal. Therefore we expect the voltage in the secondary voltage will drop more metal plates. Therefore we can extract the skin depth at frequency f from:

$$\ln V_2 = -t_0 / \delta(f) \times N + c_0 \quad (9)$$

where c_0 is a constant that we ignore. We can determine the skin depth at a frequency f using :

$$\delta(f) = -t_0 / b_1 \quad (10)$$

SOLUTION

where b_1 is the slope of $\ln(V_2)$ vs. N data.

Then we repeated this analysis at different frequencies, using Eq. 2 of the problem set:

$$\ln \delta = \frac{n}{2} \ln f + \frac{\ln(\sigma^m / \pi \mu)}{2} \quad (11)$$

Using linear model: $y = a_2 + b_2 x$ of $\ln \delta$ vs. $\ln f$, we can obtain:

$$n = 2 \frac{\Delta \ln \delta}{\Delta \ln f} = 2b_2 \quad (12)$$

The conductivity (using $m = -1$ as determined in Q2.5 later) is given as:

$$\sigma = \frac{\exp(-2a_2)}{\pi \mu} \quad (13)$$

Note: See Appendix A at the end for an alternative single regression analysis which is also valid but less accurate.

Skin Depth Experiments:

We inject the oscillating current to coil#1 and measure the induced voltage at the coil#2 while keep adding the metal pieces. The voltage in coil#2 is proportional to the magnetic field generated from coil#1 after attenuated by the metal pieces.

The student is expected to test first range of appropriate frequencies before taking data. To obtain the best results the student should perform the experiment with all 5 or 4 plates for each metal and repeat that at minimum 5 frequencies. The results are shown below.

(1) Aluminum:

	f(X)	V2(Y)	N(Y)	lnV2(Y)
Units	Hz	V	# plate	
1	1010		0	--
2	1010	0.103	1	-2.273
3	1010	0.068	2	-2.688
4	1010	0.053	3	-2.937
5	1010	0.046	4	-3.079
6	1010	0.037	5	-3.297
7	1520		0	--
8	1520	0.114	1	-2.172
9	1520	0.084	2	-2.477
10	1520	0.064	3	-2.749
11	1520	0.05	4	-2.996
12	1520	0.036	5	-3.324
13	2010		0	--
14	2010	0.129	1	-2.048
15	2010	0.081	2	-2.513
16	2010	0.057	3	-2.865
17	2010	0.046	4	-3.079
18	2010	0.036	5	-3.324
19	2660		0	--
20	2660	0.129	1	-2.048
21	2660	0.081	2	-2.513
22	2660	0.054	3	-2.919
23	2660	0.043	4	-3.147
24	2660	0.031	5	-3.474

	f(X)	V2(Y)	N(Y)	lnV2(Y)
Units	Hz	V	# plate	
25	3140		0	--
26	3140	0.16	1	-1.833
27	3140	0.093	2	-2.375
28	3140	0.063	3	-2.765
29	3140	0.046	4	-3.079
30	3140	0.034	5	-3.381
31	3540	0.342	0	-1.073
32	3540	0.16	1	-1.833
33	3540	0.107	2	-2.235
34	3540	0.08	3	-2.526
35	3540	0.049	4	-3.016
36	3540	0.034	5	-3.381
37	4000		0	--
38	4000	0.169	1	-1.778
39	4000	0.091	2	-2.397
40	4000	0.063	3	-2.765
41	4000	0.04	4	-3.219
42	4000	0.02	5	-3.912
43	4520		0	--
44	4520	0.335	1	-1.094
45	4520	0.167	2	-1.790
46	4520	0.099	3	-2.313
47	4520	0.065	4	-2.733
48	4520	0.034	5	-3.381

	f(X1)	b(Y1)	delta(Y1)	t(Y1)	lnf(X2)	ln(delta(X3))
Units	Hz	-	mm	mm	ln(Hz)	ln(m)
nents		slope			ln(f)	ln(delta)
1	1010	-0.2439	2.99303	0.73	6.918	-5.811
2	1520	-0.2824	2.58499		7.326	-5.958
3	2010	-0.3118	2.34094		7.606	-6.057
4	2660	-0.3485	2.09475		7.886	-6.168
5	3140	-0.3802	1.92026		8.052	-6.255
6	3540	-0.4395	1.66091		8.172	-6.400
7	4000	-0.5090	1.4341		8.294	-6.547
8	4520	-0.5519	1.32267		8.416	-6.628

SOLUTION

(a)

(b)

Table 6. Skin depth experiment for Al: (a) Raw data, (b) Power factor n and σ analysis.

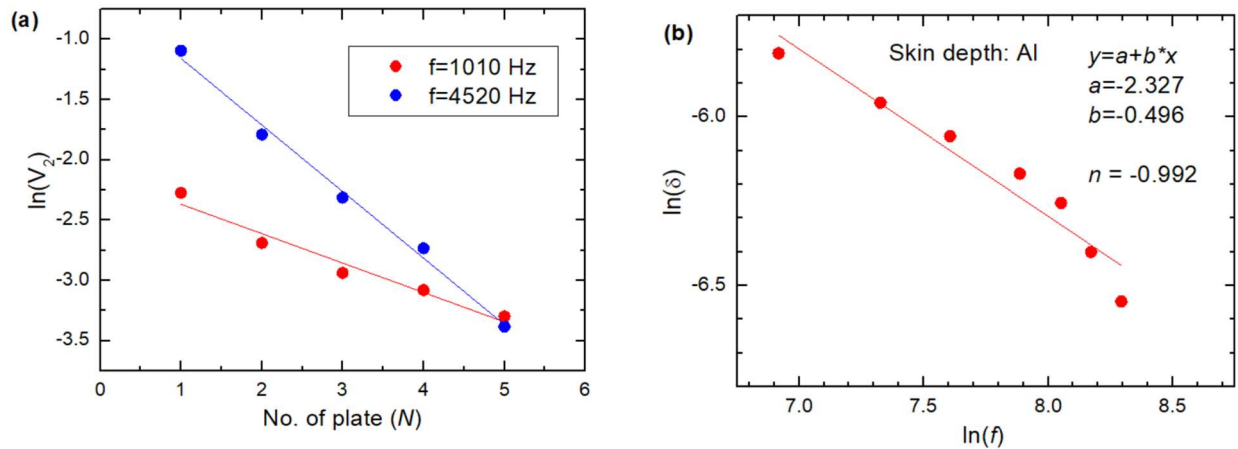


Figure 6. Skin depth analysis for Al: (a) Voltage at secondary coil vs. number of plate N at two extreme frequencies (b) Skin depth vs. frequency to determine power factor n and σ .

(2) Copper:

	f(X)	V2(Y)	N(Y)	lnV2(Y)
Units	Hz	V	# plate	
1	990		0	--
2	990	0.074	1	-2.604
3	990	0.043	2	-3.147
4	990	0.034	3	-3.381
5	990	0.026	4	-3.650
6	990	0.02	5	-3.912
7	1520		0	--
8	1520	0.074	1	-2.604
9	1520	0.043	2	-3.147
10	1520	0.031	3	-3.474
11	1520	0.02	4	-3.912
12	1520	0.014	5	-4.269
13	2030		0	--
14	2030	0.083	1	-2.489
15	2030	0.04	2	-3.219
16	2030	0.023	3	-3.772
17	2030	0.02	4	-3.912
18	2030	0.011	5	-4.510

(a)

(b)

Table 7. Skin depth experiment for Cu: (a) Raw data, (b) Power factor n and σ analysis

	f(X1)	b(Y1)	delta(Y1)	t(Y1)	lnf(X2)	ln(delta(X3))
Units	Hz	slope	mm	mm	ln(f)	ln(delta)
1	990	-0.3120	2.244	0.7	6.898	-6.100
2	1520	-0.4095	1.709		7.326	-6.372
3	2030	-0.4735	1.478		7.616	-6.517
4	2510	-0.4970	1.408		7.828	-6.565
5	3000	-0.5491	1.275		8.006	-6.665
6	4000	-0.6375	1.098		8.294	-6.814

SOLUTION

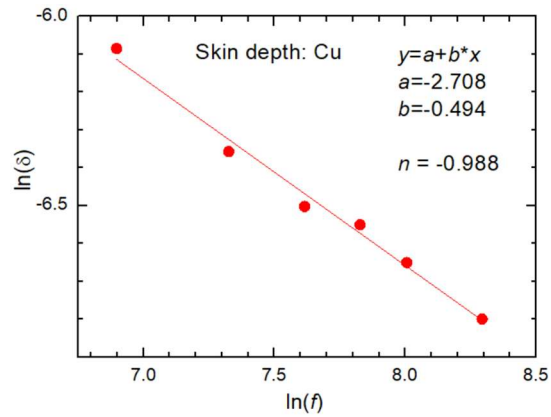


Figure 7. Skin depth analysis for Cu

(3) SS304:

Units	f(X)	V2(Y)	N(Y)	lnV2(Y)
ments	Hz	V	# plate	
1	15000		0	--
2	15000	0.61	1	-0.4943
3	15000	0.46	2	-0.7765
4	15000	0.39	3	-0.9416
5	15000	0.34	4	-1.079
6	15000		5	--
7	20000		0	--
8	20000	0.647	1	-0.4354
9	20000	0.571	2	-0.5604
10	20000	0.411	3	-0.8892
11	20000	0.342	4	-1.073
12	20000		5	--
13	25000		0	--
14	25000	0.723	1	-0.3243
15	25000	0.51	2	-0.6733
16	25000	0.392	3	-0.9365
17	25000	0.327	4	-1.118
18	25000		5	--
19	30100		0	--
20	30100	0.761	1	-0.2731
21	30100	0.518	2	-0.6578
22	30100	0.403	3	-0.9088
23	30100	0.331	4	-1.106
24	30100		5	--

(a)

Units	f(X)	V2(Y)	N(Y)	lnV2(Y)
ments	Hz	V	# plate	
25	35000		0	--
26	35000	0.79	1	-0.2357
27	35000	0.53	2	-0.6349
28	35000	0.4	3	-0.9163
29	35000	0.31	4	-1.171
30	35000		5	--
31	40000		0	--
32	40000	1.134	1	0.1258
33	40000	0.715	2	-0.3355
34	40000	0.525	3	-0.6444
35	40000	0.411	4	-0.8892
36	40000		5	--
37	45800		0	--
38	45800	1.18	1	0.1655
39	45800	0.731	2	-0.3133
40	45800	0.54	3	-0.6162
41	45800	0.42	4	-0.8675
42	45800		5	--
43	51300		0	--
44	51200	0.525	1	-0.6444
45	51200	0.297	2	-1.214
46	51200	0.217	3	-1.528
47	51200	0.194	4	-1.640
48	51200		5	--

(b)

Units	f(X1)	b(Y1)	delta(Y1)	t(Y1)	lnf(X2)	ln(delta(X3))
ments	Hz	slope	mm	mm	ln(Hz)	ln(m)
1	15000	-0.1919	3.752	0.72	9.616	-5.586
2	20000	-0.2242	3.212		9.903	-5.741
3	25000	-0.2644	2.723		10.13	-5.906
4	30100	-0.2750	2.618		10.31	-5.945
5	35000	-0.3087	2.332		10.46	-6.061
6	40000	-0.3354	2.147		10.60	-6.144
7	45800	-0.3402	2.116		10.73	-6.158
8	51200	-0.3301	2.181		10.84	-6.128

Table 8. Skin depth experiment for SS304: (a) Raw data, (b) Power factor n and σ analysis.

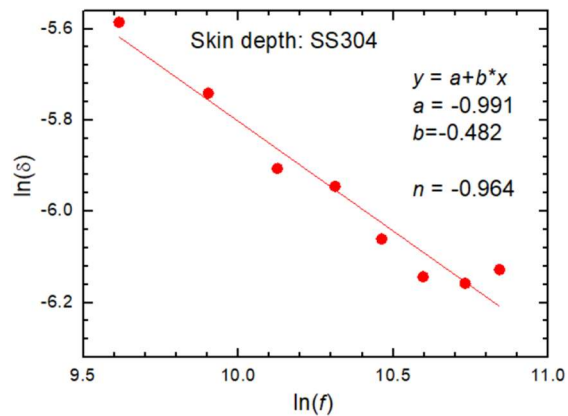


Figure 8. Skin depth analysis for SS304: Skin depth vs. frequency to determine power factor n and σ .

SOLUTION

(4) SS410:

	f(X1)	V2(Y1)	N(X2)	lnV2(Y2)
Units	Hz	V		
ents			# plate	
1	843	2.05	0	--
2	843	0.68	1	-0.3857
3	843	0.61	2	-0.4943
4	843	0.53	3	-0.6349
5				--
6	1070		0	--
7	1070	0.02	1	-3.912
8	1070	0.02	2	-3.912
9	1070	0.02	3	-3.912
10				--
11	2210	3.81	0	--
12	2210	0.99	1	-0.01005
13	2210	0.84	2	-0.1744
14	2210	0.76	3	-0.2744
15				--
16	43200		0	--
17	43200	0.753	1	-0.2837
18	43200	0.571	2	-0.5604
19	43200	0.441	3	-0.8187
20				--
21	60000	2.19	0	--
22	60000	0.36	1	-1.022
23	60000	0.2	2	-1.609
24	60000	0.14	3	-1.966
25	60000	0.1	4	-2.303
26	60000	0.07	5	-2.659

(a)

	f(X1)	b(Y1)	delta(Y1)	t(Y1)	lnf(X2)	ln(delta(X3))
Units	Hz		mm	mm		
ents		slope			ln(f)	ln(delta)
1	843	-0.1246	6.100	0.76	6.737	1.808
2	1070	--	--		6.975	--
3	2210	-0.1322	5.750		7.701	1.749
4	43200	-0.2675	2.841		10.674	1.044
5	60000	-0.3968	1.915		11.002	0.650

(b)

Table 9. Skin depth experiment for SS410: (a) Raw data, (b) Power factor n and σ analysis.

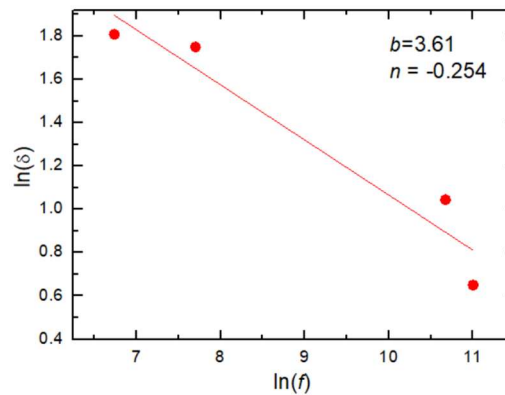


Figure 9. Skin depth analysis for SS410

We note that SS410 behaves differently, the output signal V_2 drops greatly upon insertion of the metals. Upon final analysis it yields $n = -0.5$ which is not correct.

Apparently because SS410 is magnetic, its skin depth is too small and almost no magnetic field could penetrate the metal and the fringing field around the metal becomes dominant thus the “skin depth”

SOLUTION

measurement becomes anomalous. Thus **SS410 is the metal with “extreme skin depth”** and is excluded in the subsequent analysis.

The summary of the skin depth experiment is shown below:

	Metal(X)	a(Y)	b(Y)	n(Y)	sigma(Y)	sigmaR(Y)	sigmaEr
ame							
Jnits					S/m	S/m	%
ents		intercept	slope			Ref lit.	Error
1	Al	-2.327	-0.496	-0.992	2.66E+07	3.7E7	-28.1
2	Cu	-2.708	-0.494	-0.988	5.70E+07	5.88E7	-3.1
3	SS304	-0.991	-0.481	-0.962	1.84E+06	1.39E6	32.3

Table 10. Summary results of the skin depth experiment. “sigmaErr” is the percentage error of the measured conductivity vs. the reference literature values (“sigmaR”)

We observe from all three metals we obtain consistent frequency power factor with average $n = -0.98$, and thus $n = -1$ (rounded to nearest integer).

2.5 Conductivity power factor m

Since we have obtained $n = -1$, from the Eq. 2 in the problem set:

$$\delta^2 = \frac{\sigma^m}{\pi \mu f} \quad \text{or} \quad [\sigma]^m = [\delta]^2 [\mu] [f] \quad (14)$$

Using dimensional or unit analysis: $[\sigma] = 1/\Omega \cdot \text{m} = \text{A}/\text{V} \cdot \text{m}$, $[\delta] = \text{m}$, $[\mu] = \text{H}/\text{m} = \text{V} \cdot \text{s}/\text{A} \cdot \text{m}$ and $[f] = 1/\text{s}$, we have:

$$[\text{A}/\text{V} \cdot \text{m}]^m = [\text{m}]^2 [\text{V} \cdot \text{s}/\text{A} \cdot \text{m}] [1/\text{s}] = [\text{V} \cdot \text{m}/\text{A}] \quad (15)$$

Thus we get $m = -1$, and the final skin depth formula is:

$$\delta = \frac{1}{\sqrt{\pi \sigma \mu f}} \quad (16)$$

2.6 Conductivity of the metals

The conductivity of the metals can be calculated using Eq. (13) and the results are shown in Table 10. We observe that our measured conductivity is reasonably good to the reference literature values (within +/- 30% error). The larger uncertainty is due to the results originating from a value that depends exponentially on the intercepts [Eq. (13)].

Note: This method provides a very attractive approach to perform conductivity measurement in a material because it is non-contact.

3. EXP #3: COOKING, SPECIFIC HEAT CAPACITY AND EFFECTIVE LOAD RESISTANCE

3.1 The induction cooking operating principle

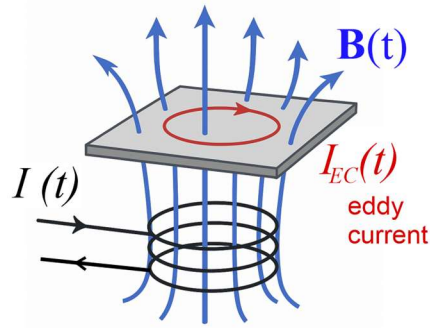


Figure 10. Principle of induction cooker

Operating principle:

Oscillating current drive to the coil → generate oscillating magnetic field → generate eddy current in the plate → generate Joule heating in the plate

3.2 Specific Heat of the metal pan

We developed a model that allow us to extract the specific heat capacity of the metal pan. The heat transfer energy balance can be modeled as total power input to the cooking pan is equal to the heating rate of the pan and radiation. We ignore convection losses as indicated in the problem.

$$P_{IN} = m c dT / dt + e A \sigma_S (T^4 - T_0^4) \quad (17)$$

where m is the mass of the metal pan, c is the specific heat, T is the plate temperature, T_0 is the ambient temperature, e is the emissivity, A is the surface area of the radiating body and σ_S is the Stefan Boltzmann constant.

We need to warm up the cooker first and then turn off the power input to let it cool. The cooling behavior is given as:

$$T^4 = -\frac{m c}{e A \sigma_S} \frac{dT}{dt} + T_0^4 = -\frac{\rho c t_0}{2 e \sigma_S} \frac{dT}{dt} + T_0^4 \quad (18)$$

We note that the factor of two comes from consideration that that the radiation area is twice the surface area of the metal i.e. $A = 2WL$, where W and L is the width and the length of the “pan”, t_0 is

SOLUTION

the metal thickness. We perform linear regression: $y = a + b x$, with x is dT/dt and y is T^4 and we can ignore the effect of T_0 .

The specific heat can be calculated as:

$$c = -\frac{2e\sigma_s b}{\rho t_0} \quad (19)$$

Note: It is possible to solve the differential equation in the Eq. (18), but the solution requires the knowledge of starting temperature T_0 which could vary with repeated experiments, thus such solution is not practical.

3.3 Specific heat of the Aluminum pan

We then measure the thermistor resistor (R_{NTC}) and calculate the pan temperature T using Eq. (4) in the problem set. Specifically, we need to derive:

$$T = \left[\frac{\ln(R/R_0)}{B} + \frac{1}{T_0} \right]^{-1} \quad (20)$$

We record ambient temperature is $T = 306.5 \text{ K} = 33.35 \text{ C}$, for completeness but this does not impact subsequent analysis. We can calculate the derivate dT/dt at point n numerically as:

$$\frac{dT_n}{dt} = \frac{T_{n+1} - T_{n-1}}{t_{n+1} - t_{n-1}} \quad (21)$$

We heat up the “pan” approximately for 1 min until the temperature reaches 325.4 K (52.3 °C) which marks $t = 0 \text{ s}$ and record the NTC resistance as a function of time as the “pan” cools.

SOLUTION

	te(X1)	RNTC(Y1)	T(Y1)	dTdt(X2)	T4(Y2)
Units	s	kOhm	K	K	K ⁴
ments	time				T ⁴
1	--	6.97	306.5	--	--
2	0	3.302	325.4	--	1.121E+10
3	20	3.532	323.6	-0.085	1.096E+10
4	40	3.753	322.0	-0.075	1.075E+10
5	60	3.961	320.6	-0.0625	1.056E+10
6	80	4.123	319.5	-0.06	1.042E+10
7	100	4.334	318.2	-0.0575	1.026E+10
8	120	4.513	317.2	-0.0475	1.012E+10
9	140	4.667	316.3	-0.0425	1.002E+10
10	160	4.818	315.5	-0.0375	9.914E+09
11	180	4.957	314.8	-0.0325	9.824E+09
12	200	5.086	314.2	-0.03	9.744E+09
13	220	5.204	313.6	-0.0275	9.673E+09
14	240	5.318	313.1	-0.025	9.607E+09
15	260	5.426	312.6	-0.0225	9.546E+09
16	280	5.519	312.2	-0.0225	9.495E+09
17	300	5.611	311.7	-0.02	9.445E+09
18	320	5.694	311.4	-0.015	9.402E+09
19	340	5.771	311.1	-0.0175	9.362E+09
20	360	5.844	310.7		9.325E+09

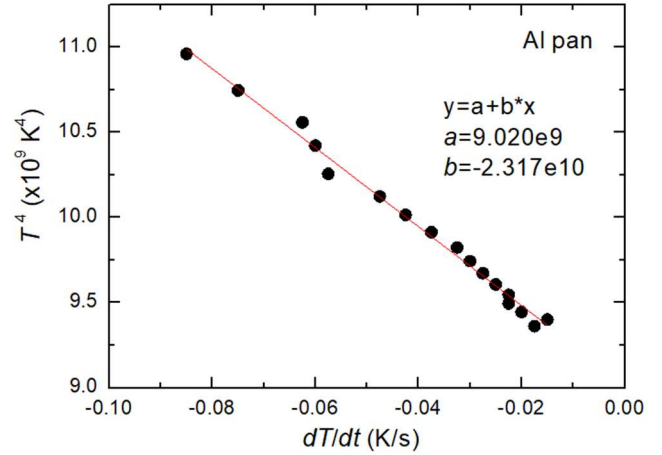


Figure 11. Specific heat measurement for Al pan

Using Eq. (19), $e = 0.65$, $\rho = 2700 \text{ kg/m}^3$, $t_0 = 0.71 \text{ mm}$, we obtain slope $b = -2.317 \times 10^{10}$ and specific heat $c = 890 \text{ J/kg.K}$. The literature value is $c_{Al} = 900 \text{ J/kg.K}$.

Note: In this Olympiad problem, the emissivity e value is chosen yield c close to the reference value.

3.4 Specific heat of the SS410 pan

	tc(X1)	RNTC(Y1)	T(Y1)	dTdt(X2)	T4(Y2)
Units	s	kOhm	K	K	K ⁴
ments					T ⁴
1		6.97	306.50	--	8.825E+09
2	0	4	320.30	--	1.053E+10
3	20	4.02	320.17	-0.02000	1.051E+10
4	40	4.127	319.49	-0.03750	1.042E+10
5	60	4.26	318.68	-0.04250	1.031E+10
6	80	4.409	317.79	-0.04500	1.020E+10
7	100	4.565	316.91	-0.04000	1.009E+10
8	120	4.703	316.15	-0.03750	9.990E+09
9	140	4.851	315.37	-0.03750	9.892E+09
10	160	4.975	314.74	-0.03250	9.813E+09
11	180	5.095	314.14	-0.02750	9.738E+09
12	200	5.205	313.61	-0.02500	9.673E+09
13	220	5.312	313.10	-0.02250	9.610E+09
14	240	5.406	312.67	-0.02000	9.557E+09
15	260	5.496	312.26	-0.02000	9.507E+09
16	280	5.58	311.88	-0.01750	9.462E+09
17	300	5.654	311.56	-0.01500	9.422E+09
18	320	5.719	311.28	-0.01500	9.389E+09
19	340	5.788	310.99	-0.01250	9.353E+09
20	360	5.843	310.75		9.325E+09

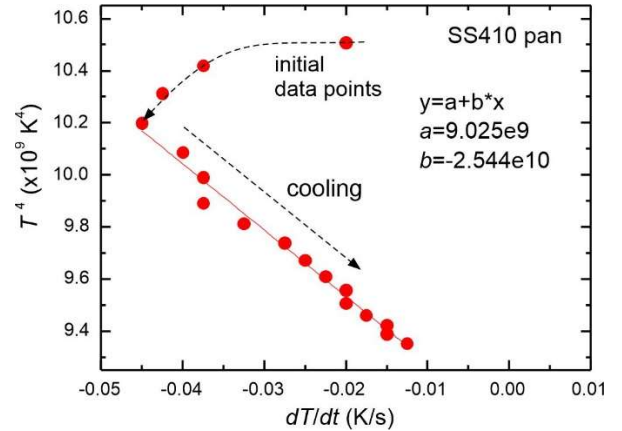


Figure 12. Specific heat measurement for SS410 pan

SOLUTION

We note sometimes, like for SS410 here, the initial data do not form a straight line as the system has not reached a steady state, thus we only perform the analysis on the linear segment as expected from the model.

Using Eq. (19), $e = 0.8$, $\rho = 7700 \text{ kg/m}^3$, $t_0 = 0.75 \text{ mm}$, we obtain slope $b = -2.544 \times 10^{10}$ and specific heat $c = 400 \text{ J/kg.K}$. The literature value is $c_{SS410} = 460 \text{ J/kg.K}$.

3.5 R_{LOAD} for Aluminum pan

Now we model that the pan appears as “load resistance” to the primary circuit. The power input given to the metal pan will increase the temperature of the “pan” and also radiate to the surrounding:

$$P_{IN} = I^2 R_{LOAD} = m c dT / dt + e A \sigma_s (T^4 - T_0^4) \quad (22)$$

For later analysis, we can rearrange this to:

$$m c dT / dt + e A \sigma_s T^4 = I^2 R_{LOAD} + e A \sigma_s T_0^4 \quad (23)$$

$$P'_{TOT} = P_C + P_{RAD}' = I^2 R_{LOAD} + P_{RAD,0} \quad (24)$$

where: $P_C = m c dT / dt$, $P'_{RAD} = e A \sigma_s T^4$ and $P_{RAD,0} = e A \sigma_s T_0^4$

We can perform linear fit of the experimental data of P_{TOT}' vs. I^2 following linear equation: $y = a + b x$, where: $y = P_{TOT}'$, $x = I^2$, $b = R_{LOAD}$ and $a = P_{RAD,0}$, which we assume to be constant and can be ignored.

Thus, we can obtain R_{LOAD} from the linear fit of P_{TOT}' vs. I^2 . Note: the current I must be of RMS value since it is an AC current.

Aluminum pan:

We now perform the “cooking” experiment on the Al “pan”. We will vary the current to the circuit and monitor the heating behavior.

	Irms(Y)	te(Y)	RNTC(X)	T(Y)	dTdt(Y)	Pc(Y)	Pradp(Y)	Ptotp(Y)	Ptotpave(Y)		Irms(Y)	te(Y)	RNTC(X)	T(Y)	dTdt(Y)	Pc(Y)	Pradp(Y)	Ptotp(Y)	Ptotpave(Y)
Units	A	A	Ohm	K	K/s	W	W	W	W		A	A	Ohm	K	K/s	W	W	W	W
ents					dT/dt	m*c*dT/dt	e*A*sB*T^4	Pc+Prad'	Ptotave						dT/dt	m*c*dT/dt	e*A*sB*T^4	Pc+Prad'	Ptotave
1	--	Ambient	--	--	--	--	--	--	--	1	--	--	--	--	--	--	--	--	--
2	0.4172	0	6.6	307.80	--	--	0.26466	--	--	2	0.4978	0	6.6	307.80	--	--	0.26466	--	0.27583
3	(0.59 pk)	20	6.546	308.00	8.704E-03	6.098E-03	0.26534	0.27143	0.27158	3	(0.704 Apk)	20	6.515	308.12	1.437E-02	1.007E-02	0.26573	0.27580	--
4	--	40	6.505	308.15	7.483E-03	5.243E-03	0.26586	0.27110	--	4	--	40	6.444	308.38	1.251E-02	8.763E-03	0.26664	0.27540	--
5	--	60	6.465	308.30	7.819E-03	5.478E-03	0.26637	0.27185	--	5	--	60	6.381	308.62	1.114E-02	7.805E-03	0.26746	0.27526	--
6	--	80	6.421	308.46	6.366E-03	4.460E-03	0.26694	0.27140	--	6	--	80	6.326	308.82	1.040E-02	7.287E-03	0.26818	0.27547	--
7	--	100	6.397	308.55	5.184E-03	3.632E-03	0.26725	0.27088	--	7	--	100	6.272	309.03	9.436E-03	6.611E-03	0.2689	0.27551	--
8	--	120	6.366	308.67	5.587E-03	3.915E-03	0.26766	0.27157	--	8	--	120	6.228	309.20	8.550E-03	5.990E-03	0.2695	0.27549	--
9	--	140	6.338	308.78	5.045E-03	3.535E-03	0.26802	0.27156	--	9	--	140	6.184	309.37	8.127E-03	5.694E-03	0.2701	0.27579	--
10	--	160	6.313	308.87	4.782E-03	3.351E-03	0.26836	0.27171	--	10	--	160	6.145	309.53	7.697E-03	5.393E-03	0.27063	0.27602	--
11	--	180	6.288	308.97	4.611E-03	3.231E-03	0.26869	0.27192	--	11	--	180	6.106	309.68	7.353E-03	5.152E-03	0.27117	0.27632	--
12	--	200	6.265	309.06	4.245E-03	2.974E-03	0.269	0.27197	--	12	--	200	6.071	309.82	6.804E-03	4.767E-03	0.27166	0.27643	--
13	--	220	6.244	309.14	3.874E-03	2.714E-03	0.26928	0.27199	--	13	--	220	6.038	309.95	6.444E-03	4.515E-03	0.27213	0.27664	--
14	--	240	6.225	309.21	--	--	0.26954	--	--	14	--	240	6.007	310.08	--	--	0.27257	--	--
15	--	--	--	--	--	--	--	--	--	15	--	--	--	--	--	--	--	--	--

SOLUTION

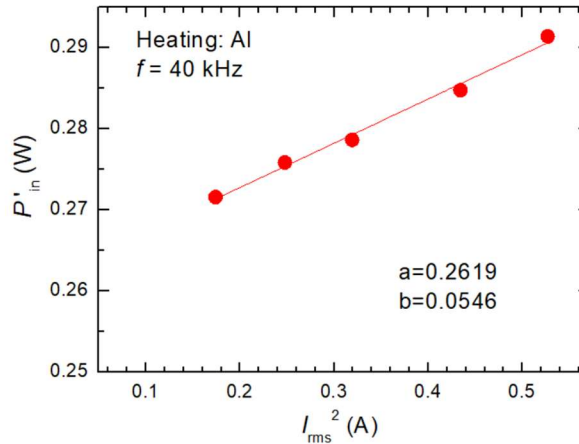
	I _{rms} (Y)	I _e (Y)	RNTC(X)	T(Y)	dTdt(Y)	P _c (Y)	Pradp(Y)	Ptotp(Y)	Ptotpave(Y)			I _{rms} (Y)	I _e (Y)	RNTC(X)	T(Y)	dTdt(Y)	P _c (Y)	Pradp(Y)	Ptotp(Y)	Ptotpave(Y)
Units	A	A	Ohm	K	K/s	W	W	W	W	Units		A	A	Ohm	K	K/s	W	W	W	W
nments					dT/dt	m ³ c ³ dT/dt	e ⁴ A ⁴ sB ⁴ T ⁴	Pc+Prad	Ptotave	nments						dT/dt	m ³ c ³ dT/dt	e ⁴ A ⁴ sB ⁴ T ⁴	Pc+Prad	Ptotave
1	0.5650	0	6.6	307.80	--	--	0.26466	--	0.27863	1	0.5650	0	6.6	307.80	--	--	--	0.26466	--	--
2	(0.799 Apk)	20	6.503	308.16	1.671E-02	1.171E-02	0.26588	0.27759	--	2	(0.932 Apk)	20	6.444	308.38	2.608E-02	1.827E-02	0.26664	0.28491	0.28476	--
3		40	6.419	308.47	1.519E-02	1.064E-02	0.26696	0.27761	--	3		40	6.32	308.85	2.213E-02	1.551E-02	0.26826	0.28377	--	--
4		60	6.341	308.77	1.398E-02	9.793E-03	0.26798	0.27778	--	4		60	6.212	309.26	2.036E-02	1.426E-02	0.26971	0.28398	--	--
5		80	6.272	309.03	1.301E-02	9.113E-03	0.2689	0.27802	--	5		80	6.111	309.66	1.845E-02	1.292E-02	0.2711	0.28403	--	--
6		100	6.206	309.29	1.209E-02	8.469E-03	0.2698	0.27827	--	6		100	6.026	310.00	1.655E-02	1.159E-02	0.2723	0.28389	--	--
7		120	6.148	309.51	1.134E-02	7.946E-03	0.27059	0.27854	--	7		120	5.947	310.32	1.578E-02	1.105E-02	0.27343	0.28448	--	--
8		140	6.091	309.74	1.076E-02	7.539E-03	0.27138	0.27892	--	8		140	5.872	310.63	1.486E-02	1.041E-02	0.27452	0.28493	--	--
9		160	6.04	309.95	1.006E-02	7.052E-03	0.2721	0.27915	--	9		160	5.804	310.92	1.370E-02	9.596E-03	0.27553	0.28513	--	--
10		180	5.991	310.14	9.549E-03	6.690E-03	0.2728	0.27949	--	10		180	5.742	311.18	1.270E-02	8.895E-03	0.27646	0.28536	--	--
11		200	5.946	310.33	8.916E-03	6.247E-03	0.27344	0.27969	--	11		200	5.685	311.43	1.198E-02	8.395E-03	0.27733	0.28573	--	--
12		220	5.904	310.50	8.369E-03	5.864E-03	0.27405	0.27992	--	12		220	5.631	311.66	1.135E-02	7.952E-03	0.27817	0.28612	--	--
13		240	5.865	310.66	--	--	0.27462	--	--	13		240	5.581	311.88	--	--	0.27895	--	--	--
14										14										
15										15										

Table 11. Data for R_{LOAD} determination of the Aluminum pan using various current.

For calculation convenience, we tabulate all the properties of the Al pan as follows:

Quantity	Symbol	Values
Emissivity	e	0.65
Mass density	ρ	2700 kg/m ³
Heat capacity	c	913.7 J/kg.K
Heat capacity reference	C_{REF}	900 J/kg.K
Radiation area	A	2x2cmx2cm = 8x10 ⁻⁴ m ²
Volume	V	2cmx2cmx0.71mm=2.84x10 ⁻⁷ m ³
Mass	m	$\rho V=7.668 \times 10^{-4}$ kg
Ambient temperature	T_0	303.66 K ($R_{NTC0} = 7.864$ k Ω)

Table 12. Properties of the Al pan



SOLUTION

Figure 13. “Effective load resistance” measurement of the aluminum pan.

We then plot $P_{\text{tot'ave}}$ vs. I_{rms}^2 as shown above. The slope directly yields the load resistance $R_{\text{LOAD}} = 54.6 \text{ m}\Omega$.

3.6 R_{LOAD} for the SS410 pan

Units	Irms(Y)	te(Y)	RNTC(X)	T(Y)	dTdt(Y)	Pc(Y)	Pradp(Y)	Ptotp(Y)	Ptotpave(Y)
	A	A	Ohm	K	K/s	W	W	W	W
					m*c*dT/dt	m*c*dT/dt	e*A*sB*T^4	Pc+Prad'	Ptotave
1	0.4441	0	6	310.11	--	--	0.33559	--	0.35215
2	(0.628 pk)	20	5.9	310.52	1.816E-02	1.805E-02	0.33737	0.35541	--
3	--	40	5.824	310.83	1.458E-02	1.448E-02	0.33875	0.35323	--
4	--	60	5.761	311.10	1.297E-02	1.288E-02	0.33991	0.35279	--
5	--	80	5.702	311.35	1.128E-02	1.121E-02	0.34101	0.35222	--
6	--	100	5.656	311.55	9.555E-03	9.493E-03	0.34189	0.35138	--
7	--	120	5.614	311.73	8.872E-03	8.815E-03	0.34269	0.35151	--
8	--	140	5.575	311.91	8.058E-03	8.007E-03	0.34345	0.35145	--
9	--	160	5.541	312.06	7.116E-03	7.070E-03	0.34411	0.35118	--
10	--	180	5.511	312.19	6.603E-03	6.561E-03	0.3447	0.35126	--
11	--	200	5.482	312.32	6.419E-03	6.378E-03	0.34528	0.35165	--
12	--	220	5.454	312.45	5.775E-03	5.738E-03	0.34584	0.35157	--
13	--	240	5.431	312.55	--	--	0.3463	--	--
14	--	--	--	--	--	--	--	--	--

Units	Irms(Y)	te(Y)	RNTC(X)	T(Y)	dTdt(Y)	Pc(Y)	Pradp(Y)	Ptotp(Y)	Ptotpave(Y)
	A	A	Ohm	K	K/s	W	W	W	W
					m*c*dT/dt	m*c*dT/dt	e*A*sB*T^4	Pc+Prad'	Ptotave
1	0.4978	0	6	310.11	--	--	0.33559	--	0.36340
2	(0.704 pk)	20	5.827	310.82	3.113E-02	3.093E-02	0.33869	0.36962	--
3	--	40	5.702	311.35	2.481E-02	2.465E-02	0.34101	0.36567	--
4	--	60	5.596	311.81	2.096E-02	2.083E-02	0.34304	0.36386	--
5	--	80	5.511	312.19	1.812E-02	1.800E-02	0.3447	0.36270	--
6	--	100	5.434	312.54	1.637E-02	1.626E-02	0.34624	0.36250	--
7	--	120	5.367	312.85	1.465E-02	1.455E-02	0.3476	0.36216	--
8	--	140	5.307	313.12	1.332E-02	1.324E-02	0.34884	0.36208	--
9	--	160	5.253	313.38	1.194E-02	1.187E-02	0.34998	0.36184	--
10	--	180	5.206	313.60	1.112E-02	1.105E-02	0.35098	0.36202	--
11	--	200	5.16	313.82	1.050E-02	1.044E-02	0.35197	0.36240	--
12	--	220	5.119	314.02	9.752E-03	9.689E-03	0.35286	0.36255	--
13	--	240	5.08	314.21	--	--	0.35372	--	--
14	--	--	--	--	--	--	--	--	--

Units	Irms(Y)	te(Y)	RNTC(X)	T(Y)	dTdt(Y)	Pc(Y)	Pradp(Y)	Ptotp(Y)	Ptotpave(Y)
	A	A	Ohm	K	K/s	W	W	W	W
					m*c*dT/dt	m*c*dT/dt	e*A*sB*T^4	Pc+Prad'	Ptotave
1	0.5650	0	6	310.11	--	--	0.33559	--	0.37036
2	(0.799 pk)	20	5.778	311.03	4.047E-02	4.021E-02	0.33959	0.37980	0.37036
3	--	40	5.616	311.73	3.121E-02	3.101E-02	0.34265	0.37366	--
4	--	60	5.492	312.28	2.569E-02	2.552E-02	0.34508	0.37060	--
5	--	80	5.387	312.75	2.261E-02	2.247E-02	0.34719	0.36966	--
6	--	100	5.295	313.18	1.990E-02	1.977E-02	0.34909	0.36887	--
7	--	120	5.217	313.55	1.776E-02	1.765E-02	0.35074	0.36839	--
8	--	140	5.146	313.89	1.635E-02	1.624E-02	0.35227	0.36851	--
9	--	160	5.082	314.20	1.499E-02	1.489E-02	0.35368	0.36857	--
10	--	180	5.024	314.49	1.382E-02	1.373E-02	0.35497	0.36870	--
11	--	200	4.971	314.76	1.273E-02	1.265E-02	0.35617	0.36882	--
12	--	220	4.923	315.00	1.122E-02	1.114E-02	0.35728	0.36842	--
13	--	240	4.883	315.20	--	--	0.35821	--	--
14	--	--	--	--	--	--	--	--	--

Units	Irms(Y)	te(Y)	RNTC(X)	T(Y)	dTdt(Y)	Pc(Y)	Pradp(Y)	Ptotp(Y)	Ptotpave(Y)
	A	A	Ohm	K	K/s	W	W	W	W
					m*c*dT/dt	m*c*dT/dt	e*A*sB*T^4	Pc+Prad'	Ptotave
1	0.7260	0	6	310.11	--	--	0.33559	--	0.40105
2	(1.027 pk)	20	5.636	311.64	6.927E-02	6.882E-02	0.34227	0.41109	--
3	--	40	5.36	312.88	5.655E-02	5.619E-02	0.34775	0.40394	--
4	--	60	5.144	313.90	4.835E-02	4.804E-02	0.35231	0.40035	--
5	--	80	4.96	314.81	4.327E-02	4.299E-02	0.35642	0.39942	--
6	--	100	4.801	315.63	3.879E-02	3.854E-02	0.36015	0.39869	--
7	--	120	4.664	316.36	3.526E-02	3.504E-02	0.3635	0.39854	--
8	--	140	4.541	317.04	3.245E-02	3.225E-02	0.36663	0.39888	--
9	--	160	4.432	317.66	2.994E-02	2.975E-02	0.3695	0.39925	--
10	--	180	4.333	318.24	2.808E-02	2.790E-02	0.3722	0.40010	--
11	--	200	4.242	318.78	2.574E-02	2.557E-02	0.37476	0.40033	--
12	--	220	4.163	319.27	2.401E-02	2.386E-02	0.37704	0.40090	--
13	--	240	4.087	319.75	--	--	0.3793	--	--
14	--	--	--	--	--	--	--	--	--

Table 13. Data for R_{LOAD} determination of the SS410 pan using various currents.

The properties of the SS410 pan:

Quantity	Symbol	Values
Emissivity	e	0.8
Mass density	ρ	7700 kg/m ³
Heat capacity	c	464.7 J/kg.K
Heat capacity reference	c_{REF}	460 J/kg.K
Radiation area	A	2x2cmx2cm = 8x10 ⁻⁴ m ²
Volume	V	2cmx2cmx0.7mm=2.8x10 ⁻⁷ m ³
Mass	m	$\rho V=2.16 \times 10^{-3}$ kg
Ambient temperature	T_0	303.66 K ($R_{\text{NTC0}} = 7.864 \text{ k}\Omega$)

Table 14. Properties of the SS410 pan

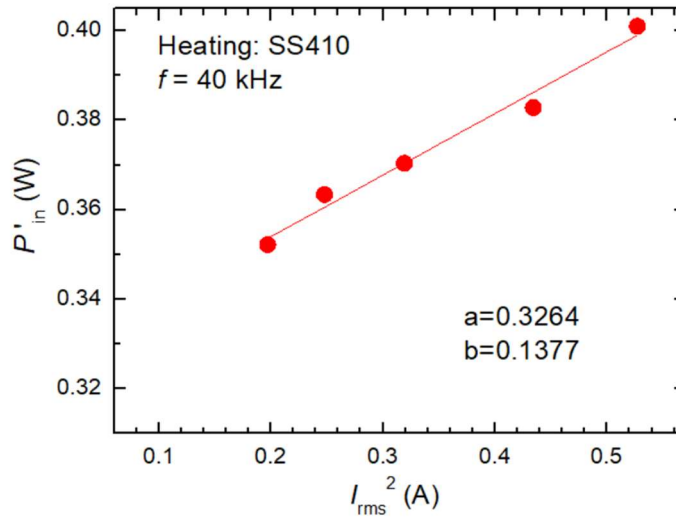


Figure 14. “Effective load resistance” measurement of the SS410 pan.

We then plot $P_{tot'ave}$ vs. I_{rms}^2 as shown above. The slope directly yields the load resistance $R_{LOAD} = 137.7 \text{ m}\Omega$ which is 2.5x than that of the Al pan.

3.7 Better cooking pan: (b) SS410.

SS410 has significantly larger R_{LOAD} (2.5x) than that of Al, thus it is more efficient to be used as induction cooking pan.

3.8 Dominant physical parameter: (b) Magnetic permeability

SS410 is a magnetic stainless steel with very high permeability $\mu_r = 700$, thus it has very small skin depth according to Eq. (16). Therefore, its R_{LOAD} is high and becomes more efficient for “cooking”.

3.9 Induction cooker efficiency:

$$\eta = \frac{P_{IND-COOK}}{P_{IN}} = \frac{I_{rms}^2 R_{LOAD}}{I_{rms}^2 (R_{LOAD} + R_L)} = \frac{R_{LOAD}}{R_{LOAD} + R_L} \quad (25)$$

From Q1.5 we have $R_L = 0.48 \text{ }\Omega$, we obtain: $\eta_{Al} = 10.2\%$ and $\eta_{SS410} = 23.4\%$. Therefore the SS410 metal is more efficient to be used as induction cooking pan.

In summary for induction cooker, we want high conductivity to allow large eddy current to be generated but very small skin-depth that could be obtained in magnetic (high permeability) material to yield higher load resistance.

Appendix:

A. Alternative Solution to skin depth analysis

For skin depth experiment we can also analyze the problem into a single linear regression analysis instead of two as shown below:

$$\ln \left[\ln \left(\frac{V_i}{V_{i+1}} \right) \right] = -\frac{n}{2} \ln f + \frac{1}{2} \ln(\pi \sigma \mu t_0^2) \quad (26)$$

where i is the index of plate used in the experiment. The conductivity can be obtained from the linear regression intercept a :

$$\sigma = \exp(2a) / \pi \mu t_0^2 \quad (27)$$

So essentially the student can perform the experiment with a single plate addition. An example of data is shown below:

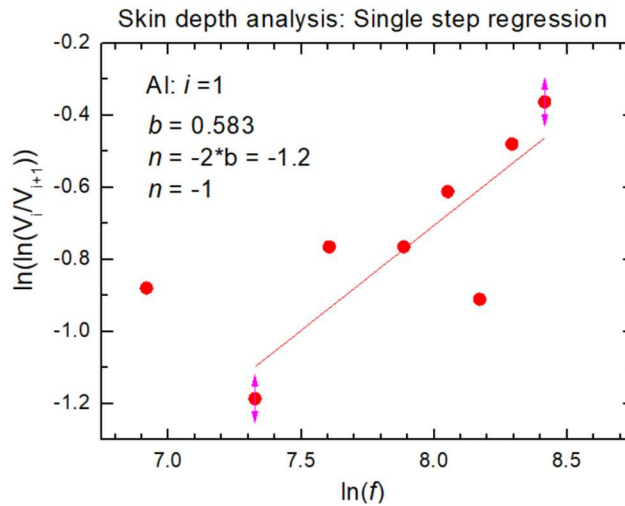


Figure 15. Single regression analysis for skin depth investigation.

The student can perform just two measurements e.g. V_2 with metal $N=1$ and $N=2$. We could also obtain $n = -1.2 \sim -1$, and $\sigma = 1.0 \times 10^7$ S/m (-73% error from reference). We observe that this technique is less accurate as it utilizes less data compared to double linear regression model that utilizes e.g. $N=5 \times 5$ frequencies data set.